INTRODUCTION

the total frequency to different class intervals, the total probability (i.e. one) is distributed to different mass points in case of a discrete random variable or to different class intervals in case of a continuous random variable. Such a probability distribution is known as Theoretical Probability Distribution.

A probability distribution also possesses all the characteristics of an observed distribution. We define mean (μ), median ($\overline{\mu}$) , mode (μ_o) , standard deviation σ etc. exactly same way we have done earlier. Again a probability distribution may be either a discrete probability distribution or a Continuous probability distribution depending on the random variable under study. Two important discrete probability distributions are

(a) Binomial Distribution and

(b) Poisson distribution.

Some important continuous probability distributions are Normal Distribution.

THEORETICAL DISTRIBUTIONS

BINOMIAL DISTRIBUTION

- Each trial is associated with two mutually exclusive and exhaustive outcomes, the occurrence of one of which is known as a 'success' and as such its non occurrence as a 'failure'.
- The trials are independent.
- The probability of a success, usually denoted by p, and hence that of a failure, usually denoted by q = 1– p, remain unchanged throughout the process.
- The number of trials is a finite positive integer.

Important points in connection with binomial distribution:

- As n >0, p, q $\geq$ 0, it follows that f(x) $\geq$ 0 for every x.
- Binomial distribution is known as biparametric distribution as it is characterised by two parameters n and p.
- The mean of the binomial distribution is given by μ
- Depending on the values of the two parameters, binomial distribution may be unimodal or bi- modal.

- N O , the mode of binomial distribution, is given by μ_o = the largest integer contained in (n+1)p if (n+1)p is a non-integer (n+1)p and (n+1)p – 1.
- The variance of the binomial distribution is given by σ^2 = npq.

POISSON DISTRIBUTION

Poisson distribution is a theoretical discrete probability distribution which can describe many processes.

Poisson Model Let us think of a random experiment under the following conditions:

I. The probability of finding success in a very small time interval (t, t + dt) is kt, where k (>0) is a constant.

II. The probability of having more than one success in this time interval is very low.

III. The probability of having success in this time interval is independent of t as well as earlier successes.

THEORETICAL DISTRIBUTIONS

Application of Poisson distribution

- The distribution of the no. of printing mistakes per page of a large book.
- The distribution of the no. of road accidents on a busy road per minute.
- The distribution of the no. of radio-active elements per minute in a fusion process.
- The distribution of the no. of demands per minute for health centre and so on.

NORMAL OR GAUSSIAN DISTRIBUTION

If the probability density function of the random variable x is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} . e^{-(\bar{x}-u)^2/2\sigma^2}$$

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THEORETICAL DISTRIBUTIONS

Some important points relating to normal distribution are listed below:

a) The name Normal Distribution has its origin some two hundred years back as the then mathematician were in search for a normal model that can describe the probability distribution of most of the continuous random variables.
b) If we plot the probability function y = f (x), then the curve, known as probability curve,
c) If we take $\mu = 0$ and $\sigma = 1$.
d) The normal distribution is known as biparametric distribution as it is characterized by two parameters μ and σ^2 . Once the two parameters are known, the normal distribution is completely specified.

Standard Normal Distribution:

If a continuous random variable z follows standard normal distribution, to be denoted by z ~ N(0, 1), then the probability density function of *z* is given by

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

Some important properties of z are listed below:

- z has mean, median and mode all equal to zero.
- The standard deviation of z is 1. Also the approximate values of mean deviation and quartile deviation are 0.8 and 0.675 respectively.
- The standard normal distribution is symmetrical about z = 0.
- The two points of inflexion of the probability curve of the standard normal distribution are –1 and 1.
- The two tails of the standard normal curve never touch the horizontal axis.

$$Z = \frac{\sqrt{n}(\bar{x} - \mu)}{\sigma} \sim N(0, 1)$$